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Etnomatemática de *Damaru*: Etnomodelagem sob Perspectivas Local (Êmica) e Global (Ética)

Ethnomathematics of *Damaru*: Ethnomodelling from Local (Emic) and Global (Etic) Perspectives

Etnomatemáticas de *Damaru*: Etnomodelación desde perspectivas locales (Émica) y globales (Ética)

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Resumo

Etnomatemática é o estudo de conceitos e práticas matemáticas visto que estão inerentemente incorporados e manifestados nos objetos culturais e nas atividades diárias de várias comunidades. Este estudo investiga os sofisticados conceitos matemáticos incorporados na construção de artefatos culturais como Damaru, pela comunidade Chundara. O Damaru é um tambor bilateral produzido pela comunidade Chundara. O Damaru serve como estudo de caso para ilustrar a integração da etnomatemática e da etnomodelagem. Esta investigação destaca como os artesãos Chundara utilizam conhecimentos matemáticos implícitos no seu trabalho artesanal, que inclui princípios geométricos e físicos. Ao aplicar perspectivas êmicas (internas) e éticas (externas), o estudo estabelece uma ponte entre práticas culturais e matemática formal. O estudo revela que conceitos matemáticos como comprimento, largura, altura, área, volume e muito mais são essenciais para as atividades e práticas diárias dos Chundaras. Estes sistemas de conhecimento, derivados dos seus antepassados fora da escolaridade formal, são factuais, científicos e fundamentais para o seu trabalho e vida.

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Palavras-chave: Artefatos culturais. Etnografia. Etnomodelagem. Conhecimento êmico. Conhecimento ético.

Abstract

Ethnomathematics is the study of mathematical concepts and practices as they are inherently embedded and manifested in the cultural objects and daily activities of various communities. This study investigates the sophisticated mathematical concepts embedded in the construction of cultural artefact such as Damaru, by the Chundara community. The Damaru is a two-sided drum produced by the Chundara community. The Damaru serves as a case study to illustrate the integration of ethnomathematics and ethnomodelling. This research highlights how Chundara artisans use implicit mathematical knowledge in their craftsmanship, which includes geometric and physical principles. By applying both emic (insider) and etic (outsider) perspectives, the study bridges cultural practices with formal mathematics. The study reveals that mathematical concepts such as length, breadth, height, area, volume, and more are integral to the Chundaras' daily activities and practices. These knowledge systems, derived from their ancestors outside formal schooling, are factual, scientific, and foundational for their work and living.

Keywords: Cultural artifacts. Ethnography. Ethnomodelling. Emic knowledge. Etic knowledge.

Resumen

La etnomatemática es el estudio de conceptos y prácticas matemáticas tal como están inherentemente integrados y manifestados en los objetos culturales y las actividades diarias de diversas comunidades. Este estudio investiga los sofisticados conceptos matemáticos incorporados en la construcción de artefactos culturales como Damaru, por parte de la comunidad Chundara. El Damaru es un tambor de dos caras producido por la comunidad Chundara. El Damaru sirve como estudio de caso para ilustrar la integración de la etnomatemática y el etnomodelado. Esta investigación destaca cómo los artesanos Chundara utilizan el conocimiento matemático implícito en su artesanía, que incluye principios geométricos y físicos. Al aplicar perspectivas tanto émicas (internas) como éticas (externas), el estudio une las prácticas culturales con las matemáticas formales. El estudio revela que conceptos matemáticos como largo, ancho, altura, área, volumen y más son parte integral de las actividades y prácticas diarias de los Chundaras. Estos sistemas de conocimiento, derivados de sus antepasados fuera de la escolarización formal, son objetivos, científicos y fundamentales para su trabajo y su vida.

Palabras clave: Artefactos culturales. Etnografía. Etnomodelación. Conocimiento émico. Conocimiento ético.

Nepalese Society for Ethnomathematical Studies (NSEmS): Activities and Future Directions

The Nepalese Society for Ethnomathematical Studies (NSEmS) was established with the aim of advancing research in ethnomathematics. NSEmS also recognizes the importance of equipping teachers with the knowledge and skills necessary to incorporate ethnomathematics into their pedagogy. This interdisciplinary fields aims to highlight the

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diverse mathematical practices rooted in various cultural traditions, thereby promoting a more inclusive view of mathematics that respects and values cultural diversity.

NSEmS's objectives are closely aligned with those of the International Study Group on Ethnomathematics (ISGEm), an organization dedicated to fostering global collaboration in ethnomathematical research. By aligning with ISGEm, NSEmS positions itself as a crucial player in the global ethnomathematics community, contributing to the broader goals of cultural inclusivity and educational equity. It is important to state here that the authors of this article are members of NSEmS, as well as they help this society to reach this goal nationally and internationally.

Despite its ambitious goals, NSEmS is currently in its infancy. The organization is in the process of establishing itself and building momentum for its activities. Progress has been gradual, reflecting the challenges inherent in developing a new research field in a country where ethnomathematics is still gaining recognition. However, the society's foundational efforts have laid the groundwork for future growth and development. One of the significant milestones for NSEmS was the organization of the First National Symposium on Ethnomathematics, held on November 9, 2019. Figure shows participants of the First National Symposium on Ethnomathematics.

Figure 1: Participants of the First National Symposium on Ethnomathematics

Source: Author's personal file

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This event was a collaborative effort with the Department of Mathematics Education at Mahendra Ratna Campus, Tribhuvan University, Kathmandu, Nepal. The symposium marked a crucial step in raising awareness about ethnomathematics in Nepal and fostering a community of researchers and educators interested in this field. The keynote speaker for the symposium was Prof. Dr. Daniel Clark Orey from the University of Ouro Preto, MG, Brazil, a renowned figure in the field of ethnomathematics.

Dr. Orey's presence added considerable prestige to the event and provided valuable insights into the global landscape of ethnomathematical research. His address likely inspired participants by illustrating the rich potential of ethnomathematics to transform mathematics education and research. More than 200 teacher educators, researchers, and students from various universities had participated in the symposium that highlighted the growing interest in ethnomathematics within Nepal.

The Nepalese Society for Ethnomathematical Studies (NSEmS) is making significant strides in promoting ethnomathematics within Nepal and internationally. One of its key contributions is its active role in organizing the Seventh International Conference on Ethnomathematics (ICEm-7). Figure 2 shows the promotional material of ICEm-7.

Figure 2: Promotional material of ISGEm-7

Source: ICEm-7

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This collaboration not only highlights NSEmS's growing influence in the global ethnomathematics community but also underscores its commitment to fostering international dialogue and exchange in this field. Looking ahead, NSEmS is set to organize its First National Conference on Ethnomathematics on December 8-9, 2024. This event aims to further strengthen the society's role in the national academic landscape and provide a dedicated platform for research scholars, educators, and students to discuss and explore ethnomathematics in the context of Nepal. NSEmS has set up various programs aimed at both in-service and prospective teachers.

These initiatives are designed to provide comprehensive training and resources, enabling educators to integrate ethnomathematical concepts into their teaching practices effectively. NSEmS employs a variety of means and programs to achieve its educational goals. Workshops, seminars, and professional development courses are organized to provide hands-on training and theoretical knowledge. The Nepalese Society for Ethnomathematical Studies (NSEmS) is expected to work closely with the International Study Group on Ethnomathematics (ISGEm) and similar professional organizations. This collaboration will help NSEmS's strategy for advancing ethnomathematics research and education both within Nepal and internationally.

At the final, the Nepalese Society for Ethnomathematical Studies is playing a pivotal role in advancing ethnomathematics both within Nepal and on the international stage. Through its involvement in major conferences, organization of national events, and dedicated teacher training programs, NSEmS is fostering a deeper understanding of the cultural dimensions of mathematics and promoting a more inclusive approach to mathematics education.

In this context, we highlight that ethnomathematics is the study of mathematical concepts and practices that are inherently embedded and manifested in the cultural objects and daily activities of various communities. The study presented in this article investigates the sophisticated mathematical concepts embedded in the construction of cultural artefact such as Damaru, by the Chundara community. The Damaru is a two-sided drum produced by the Chundara community. The Damaru serves as a case study to illustrate the integration of

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ethnomathematics, modelling, and ethnomodelling, which is in accordance to the objctives and cultural perspectives of NSEmS.

In this regard, as NSEmS continues to grow and expand its activities, it is poised to make significant contributions to the field of ethnomathematics and enhance the educational landscape in Nepal.

Ethnomathematics and Cultural Artifacts

The term ethnomathematics has been defined from various perspectives. According to D'Ambrosio (2006), "Ethnomathematics is the mathematics practiced by cultural groups, such as urban and rural communities, groups of workers, professional classes, children in a given age group, indigenous societies, and many other groups identified by their common objectives and traditions" (p. 1). According to Rosa and Orey (2010), ethnomathematics refers to the mathematical ideas and concepts embedded within diverse cultural contexts. Ascher and Ascher (1997) define ethnomathematics as the study of the mathematical ideas of non-literate people. Ethnomathematics focus on paying attention to the students' shared every day experience (Cimen, 2014).Others, such as Ferreira (1989), view it as the methodological approaches in the process of learning mathematics.

In accordance to this context, D'Ambrosio (2006) describes ethnomathematics as a research program focused on the history and philosophy of mathematics, as well as the ways in which cultural groups understand, articulate, and utilize concepts and practices that we describe as mathematical, regardless of whether these groups have developed a formal concept of mathematics. Bishop (1991) posits that mathematics is a cultural product that emerges from various activities such as counting, locating, measuring, designing, playing, and explaining. Thus, ethnomathematics refers to the mathematical concepts inherent in and practiced by the culture of a particular group of people. To enhance ethnomathematics in the teaching learning process, we used the mother tongue as the medium of instruction and incorporated local knowledge when framing curriculum and teaching practices (Acharya et al., 2021).

In Nepal, ethnomathematics is a relatively new field of research associated with mathematics education. It is described as the study of mathematical ideas and activities within

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their cultural context. Everyday life is deeply embedded with the knowledge and practices of a culture. Individuals from all cultures engage in activities like comparing, classifying, quantifying, measuring, explaining, inferring, generalizing, and evaluating, using materials and intellectual tools inherent to their culture (D'Ambrosio, 2006). The cultural activities invented by diverse peoples invariably incorporate some form of mathematical knowledge and concepts.

The Chundara people, as a caste and worker group, have their unique ways of understanding and applying mathematical knowledge. The mathematical concepts they practice in their everyday lives are often overlooked by traditional school curricula, yet these concepts are known as ethnomathematics. The Chundara possess a wealth of mathematical knowledge and ideas that they perform uniquely, often in unspoken and informal ways embedded in their workplace practices. This group of people has ethnoknowledge, which is acquired through culturally relevant teaching and learning processes (Orey; Rosa, 2010; Pradhan, 2017). Building synergy, the hybridization of indigenous knowledge and imported western knowledge can promote meaningful and a broader understanding by co-constructing knowledge base on diverse epistemological perspectives (Rai; Acharya, 2021).

Ethnomathematics can be described as the study of mathematical ideas and activities embedded within their cultural context. D'Ambrosio (2006) defines ethnomathematics as "the mathematics practiced among identifiable cultural groups such as national-tribe societies, labor groups, children of certain age brackets, and professional classes" (D'Ambrosio, 2006, p. 44). Barton (1996) further describes ethnomathematics as the way people from a particular culture have common systems for managing the quantitative, relational, and spatial aspects of their lives. Knijnik (1997) describes an ethnomathematical approach as one that investigates the traditions, practices, and mathematical concepts of a subordinated social group and develops pedagogical methods to help the group interpret and decode its knowledge.

The ethnomathematical ideas of various cultural groups have generally been excluded from formal and academic mathematics discussions. Rosa and Gavarrete (2017) also note that the mathematical knowledge and learning approaches of these groups are not considered in formal school mathematics curricula. Integrating the mathematical ideas from working-class cultures, acknowledging their knowledge generation and transmission methods, and blending

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students' experiences with formal mathematics in classrooms is essential (Pradhan, 2017). By creating and integrating mathematical materials related to different cultures and drawing on students' experiences, it is possible to apply ethnomathematical strategies in teaching and learning mathematics (Rosa; Gavarrete, 2017).

Cultural artifacts are objects created by a specific group of people that help define their culture. Gueudet and Trouche (2009) expand this definition by introducing the term "resources" to include any artefact that has the potential to enhance the learning process. Various types of cultural artifacts reflect the cultural identity of different groups, including dress, houses, utensils, baskets, ornaments, paintings, and designs. These artifacts provide insights into the culture of their creators and the ideas linked to these objects. There is a lack of concrete evidence pinpointing the exact date when people worldwide began weaving items.

However, for thousands of years, various cultures have engaged in the practice of weaving, creating baskets, hats, mats, and other items. Different cultures have utilized a wide array of materials for weaving, such as grass, leaves, cloth, feathers, paper, and other locally available resources. An artefact, defined as any human creation, serves different purposes across diverse cultures and reflects the unique needs and practices of its creators.

Cultural artifacts serve as powerful tools for enhancing the knowledge and understanding of mathematical concepts, as well as recognizing the connections between mathematics and the real world around us. The extensive use of cultural artifacts in school mathematics makes the subject more meaningful and relatable. When introduced into the mathematics classroom, these cultural artifacts act as concrete materials that children typically encounter in real-life situations (Bonotto, 2007; Pradhan, 2021).

These tangible materials can effectively transfer knowledge from one domain to another. Cultural artifacts, therefore, hold immense potential as tools for communicating abstract mathematical ideas (Pradhan, 2019). For instance, money of different denominations serves specific purposes within various communities but can also be used metaphorically in an educational context to teach and learn different mathematical concepts. In this study, I have collected several cultural artifacts that have significant potential as teaching and learning tools for conveying abstract mathematical ideas.

This paper specifically explores the mathematical concepts embedded in cultural

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artifacts such as the *Damaru*, a wooden drum, from the perspective of its creators and its scientific knowledge. It also examines its use as an instructional material that facilitates the teaching and learning of school mathematics. *Damaru* is a two sided drum like musical instrument popular in Hinduism. In Hinduism, the damru is known as the instrument of the deity *Shiva*, and is said to be created by *Shiva* to produce spiritual sounds by which the whole universe has been created and regulated. The hollow part of the instrument is the product of the Chundaras.

Thus, ethnomathematical ideas embedded in cultural artifacts help conceptualize abstract mathematical ideas by connecting to children's everyday activities. This paper argues that cultural artifacts and their embedded ethnomathematical ideas and concepts facilitate the understanding of school mathematics.

Emic and Etic Perspective in Ethnomodelling

The use of mathematical language, ideas and concepts in the process of elaboration of different cultural artifacts is known as ethnomodelling. The term ethnomodelling is relatively new term in the domain of mathematics education research. Ethnomodelling is the study of mathematical ideas and procedures developed by members of distinct cultural groups, focusing on the mathematical practices used and presented in various daily life situations by these groups (Rosa & Orey, 2010). Ethnomodelling enable participants to study mathematics as a system rooted in their own reality, striving to understand all components of these systems and their interrelationships (D'Ambrosio, 1993; Rosa & Orey, 2013).

Through ethnomodelling, researchers have uncovered sophisticated mathematical ideas and practices, including geometric principles in craft work, architectural concepts, and activities and cultural artifacts of various indigenous community (Eglash et al., 2006). These concepts are associated with numerical relations found in measuring, calculation, games, divination, navigation, astronomy, modelling, and numerous other mathematical procedures and cultural artifacts (Eglash et al., 2006). In this context, Rosa and Orey, (2010) expand the concept ethnomodelling as the intersection of cultural anthropology, ethnomathematics, and mathematical mode[ling. It serves as a pedagogical tool within an ethnomathematics program, enabling students to learn how to engage with authentic situations and real-life problems.

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Researchers and investigators need to be aware of their own theoretical and ideological biases and aim to accurately capture and represent the mathematical knowledge inherent in the work being modeled. This includes translating the significance of the work from the perspective of insiders (emic) to that of outsiders (etic).

Ethnomodelling emphasizes the organization and presentation of mathematical ideas and procedures developed by distinct cultural groups, facilitating their communication and transmission across generations. Members of these groups create ethnomodels of mathematical practices found in sociocultural systems (Rosa; Orey, 2010; Sharma; Orey, 2017), thus linking cultural heritage with the evolution of mathematical practice. This approach supports the organization of pedagogical actions in classrooms by integrating the emic and etic aspects of mathematical knowledge, enhancing the educational experience by grounding it in cultural context.

The emic perspective in research sees the reality of cultural elements in terms of the insider's perspective. In this regards, Rosa and Orey (2012) viewed that the emic approach seeks to understand a particular culture based on its own reference. Further, Fetterman (2010) said that "the insider's perception of reality is instrumental to understanding and accurately describing situations and behaviors" (Further; Fetterman, 2010, p. 20). The emic method is used to investigate how the members of particular cultural groups think, how they perceive, how they explain things and what it means for them. Thus, emic perspectives and interpretations within a culture are determined by local custom, meaning, and belief and best described through the lens of the indigenous culture. The emic approach to research in mathematical ideas in Chundara culture investigated mathematical phenomena and their interrelationships through their lens, their perspective. In this vein, Orey and Rosa (2015) mentioned that the primary goal of an emic approach in ethnomathematical research is to emphasize the uniqueness of mathematical ideas, procedures, and practices developed by the members of particular cultural groups.

An etic perspective in research views cultural elements through the eyes of the outsider. Etic knowledge is a description of a behavior or belief of the members of cultural groups by a scientific observer through the culturally neutral and outsider's perspective. The etic perspectives investigate lawful relationships and casual explanations valid across different

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cultures (Orey; Rosa, 2015). Thus the etic method applies an outsider's interpretation of events, customs, beliefs from decontextualized perspectives (Neuman 2008). Further, an etic approach is a way of examining the emic knowledge of the members of cultural groups and it is universal in the international community of scientific observers, researchers and investigators (Orey; Rosa, 2015; Sharma; Orey, 2017; Pradhan, 2017).

Fetterman (2010) highlighted that the best ethnography requires both emic and etic perspectives. The emic approach to research tries to apply overarching values to a single culture from native's lens. However, the etic approach helps in enabling researchers to see more than one aspect of a culture and to apply the observations to cultures around the globe. Thus, by combining these two approaches in study, a richer view of cultural elements of group members can be better understood. In this research both emic and etic perspectives were used to increase understanding of the knowledge generation and distribution, and the ethnomathematical ideas embedded in Chundara cultural setting.

Mathematical Manifestations in *Damaru:* Emic and Etic Perspective

Mathematical thinking and methods vary significantly across different cultures (Gerdes, 1997). For instance, the techniques used by rural communities in Nepal to construct the framework for weaving mats exhibit mathematical principles similar to those employed by Mozambican peasants in building rectangular house bases (Gerdes, 1999). Furthermore, Gerdes (1997) demonstrated that the construction methods used by house builders can be linked to the formulation of geometric axioms, such as the rectangle axiom. In examining the mat-weaving process in the rural region of Nepal, similar results were obtained and it becomes evident that it encompasses a rich array of mathematical ideas witin the indigenous communities of different parts of the world (Pradhan, 2023).

This traditional practice involves intricate calculations and spatial reasoning, reflecting complex mathematical concepts that can be harnessed in educational contexts. For example, the geometric patterns and measurements involved in mat weaving highlight principles of symmetry, proportion, and area. Incorporating indigenous mat weaving into the mathematics curriculum offers a valuable opportunity to connect abstract mathematical concepts with tangible, culturally relevant activities. By integrating such traditional practices into teaching,

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educators can bridge the gap between students' cultural experiences and formal mathematical education. This approach not only enriches the learning experience but also validates and utilizes the mathematical knowledge embedded in students' cultural backgrounds, making mathematics more accessible and meaningful from primary through higher education levels.

The *Damaru* is a two-sided drum-like musical instrument that holds significant cultural importance in Hinduism. Revered as the instrument of the deity Shiva, it is believed that Shiva created the damru to produce spiritual sounds that were instrumental in the creation and regulation of the entire universe. The hollow section of this instrument is crafted by the Chundaras. The frequency of the vibrations depends on the mechanical force applied, area of cross-sectional part i.e., surface of the base face of the parabolic cylinder and the length of cylinder. Higher the pressure of the mechanical force and lesser the cross- sectional area, greater will be the frequency. Sound waves of only those frequencies sustain, with which the air column can vibrate. Sound waves of other frequencies die out. So, the parabolic cylinder (organ pipe) can produce notes of definite frequencies. As an ethnomathematical researcher, I explored the mathematical meaning of the materials they treasured from both emic and etic perspective.

Emic Ethnomodelling of *Damaru*

Among the various wooden stuffs, *Damaru* is one of the exclusive product of Chundaras. Before preparing it, a cylindrical wooden timber is required. The whole process of making a hollow part of *Damaru* was observed. It involves lots of mathematical knowledge from the timber cultivation to the end process of constructing *Damaru.* The piece of timber for *Damaru* is in the form of a circular cylinder in shape, and the base face is the circle. The centre of the base is attached with the axial which is rectangular parallelepiped in shape. Chundara has knowledge about the circular cylinder and the rectangular parallelepiped. By managing the good combination of rotation of the machine and the *Bako,* the artisans give the required shape and size to the *Damaru*. The base face of the *Damaru* has a number of concentric circles. They also have good knowledge about the area of the circle. Regarding the *Damaru* and its shape, one of my research participants speaks:

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> *The surfaces of two ends of the Damaru are equal and do have the similar sounds from the both ends if the leashes strike the drum heads. However, we try to make variations in the end-surfaces of the Damaru so that the sound produced from the bigger surface is quite different from the smaller one.*

This illustrates that the sound is determined by the length, diameter of the end and the mechanical force given to play the instrument. This shows the presence of physical theory related to the open and enclosed organ pipe in the indigenous knowledge of the Chundaras. Rattan straps with knots on both ends strike the drum heads as it is shaken producing a rattling sound. The focus on uniformity and smoothness highlights an implicit understanding of geometric principles. The Chundara people intuitively apply concepts of equality and balance, which are crucial for the *Damaru*'s acoustical properties. An uneven or inconsistent hollow would result in a distorted sound, indicating that their traditional methods are designed to ensure that the drum produces a harmonious tone.

The Chundara people have a traditional process for crafting the *Damaru*, a two-sided drum, from timber logs. They select timber using nonstandard measuring units, such as handspans and digits (amal), to gauge the appropriate size and shape (Pradhan, 2017). By applying these measurements, they meticulously shape the timber to achieve the desired form of the *Damaru*. Their method reflects an intricate understanding of geometry and material properties, despite not relying on formal measurement tools. I asked another research participant regarding the construction process of hollow section of *Damaru*.

> *To start with the Damaru, we need to choose the right piece of timber. I use my handspan to gauge the length we need. It's a bit rough, but it works for us. When carving, we must be careful to maintain symmetry. If one side is uneven, the drum won't produce the balanced sound we need. I always check the thickness of the walls regularly.*

From an ethnomathematics perspective, ensuring that the hollow part of the *Damaru* is smooth and consistent involves a deep understanding of the mathematical principles embedded in traditional craftsmanship. The Chundara people's approach to carving the *Damaru* demonstrates an intuitive grasp of symmetry, proportion, and uniformity, even if they do not explicitly label these concepts with formal mathematical terminology. The process of achieving a uniform hollow involves practical applications of measurement and estimation techniques passed down through generations.

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The use of non-standard units, such as handspan and digits (amal), reflects a culturally specific method of quantifying and controlling dimensions. These units, while informal compared to standardized measurements, are highly effective within the context of their craft, showing an ingrained sense of precision and calibration that is essential for producing a quality instrument. In this context, the ethnomathematical knowledge embedded in their practices reveals a sophisticated alignment with mathematical concepts of symmetry and proportion. The craftsmanship of the *Damaru* is a testament to how cultural practices integrate mathematical thinking, demonstrating that traditional methods are not merely empirical but are underpinned by a coherent, albeit culturally specific, mathematical framework.

The Chundara people's construction of the *Damaru* drum showcases a rich tapestry of ethnomathematical ideas rooted in their traditional practices. They utilize nonstandard measurement units to select and prepare timber, reflecting their practical and culturally specific approach to measurement. In shaping the *Damaru*, they apply implicit geometric principles, crafting a cylindrical form with a hollow interior that demonstrates their intuitive understanding of volume and surface area. Their construction process involves precise attention to proportionality and symmetry, ensuring that both sides of the drum are balanced and function correctly. The Chundaras exhibit advanced estimation skills in timber selection and shaping, highlighting their practical knowledge of material properties. This process not only integrates mathematical concepts but also embeds them within the cultural and spiritual significance of the *Damaru*, illustrating how traditional knowledge and mathematical understanding are seamlessly intertwined in their craftsmanship.

Etic Ethnomodelling of *Damaru*

The etic ethnomodel attempts to investigate and understand phenomena and their structural interrelationship of the members of the *Chundara* community and mathematical procedures through the eyes of scientific mathematical knowledge (Sharma; Orey, 2017). *Damaru* is a musical instrument having cylindrical shape with congruent parallel circular bases. *Damaru* has rich opportunity to communicate geometrical ideas of circle, cylinder, cone, parabola, and paraboloid surface. The *Damaru* can be one of the powerful teaching aids to conceptualize paraboloid surface.

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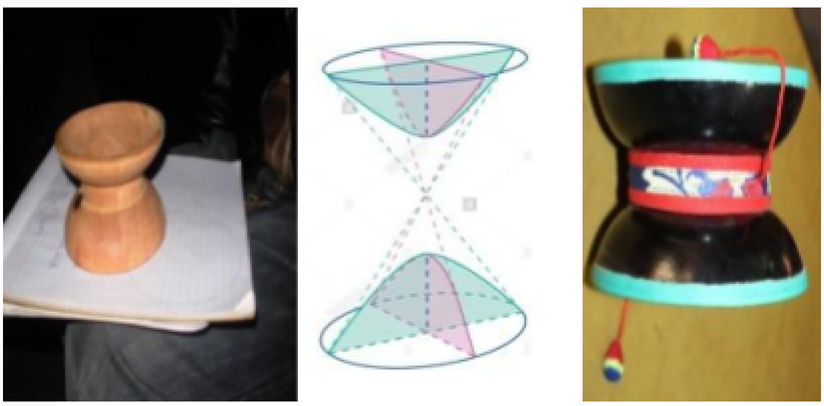
Damaru is one of the exclusive product of Chundaras which involves large number of geometrical concepts. As an ethnomathematical researcher, I have to give the mathematical meaning of the materials they treasured. Before preparing it, a cylindrical wooden timber is required. If we take one face of the *Damaru* as the base and the centre of the other face as the vertex, it will be of the form of a right circular cone. They have the master of geometry as their artifacts are full of the geometrical shape and size.

They can also be considered as the experts of ethno science as they do the best combination for the shape size and the sound produced by it. They have good knowledge between the diameter of the base face of the *Damaru* and the circumference of it. Mathematically modelling the *Damaru* drum involves understanding its geometric structure and translating this into a set of equations that describe its shape.

The *Damaru* drum typically has a shape that resembles two conical frustums joined at their bases. Each frustum is a truncated cone, and the overall shape can be approximated as a double-conical structure. A *Damaru* is typically a double-conical or drum-like instrument with a roughly cylindrical or conical shape, rather than a hyperboloid. However, if we consider the general shape of the *Damaru*, it features a central cylindrical or conical body with rounded ends, which might loosely resemble some of the geometric properties of hyperboloids in a very abstract sense. The *Damaru*'s form is more about practical functionality and acoustics rather than precise geometric modelling. Figure 3 shows the Damaru as a double conical drum.

Figure 3: The Damaru as a double-conical drum



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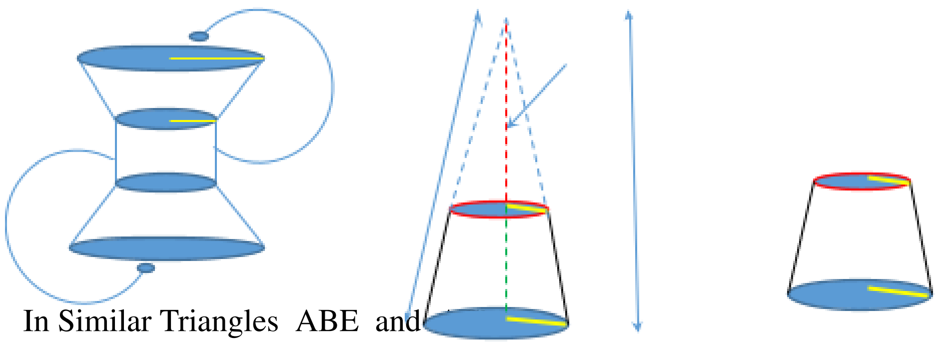
When formal school mathematics teachers and students closely observe the Damaru, which consists of two truncated cones connected at their bases with a central cylindrical section, they can explore the concepts of surface area and volume in a tangible way. The surface area of each truncated cone can be calculated by understanding the curved surfaces and the circular ends, while the cylinder's surface area is determined by its height and radius. Additionally, the total volume of the Damaru can be derived by summing the volumes of the two truncated cones and the central cylinder. This hands-on observation not only reinforces geometric principles but also connects abstract mathematical concepts to a real-world cultural artifact, deepening students' understanding of both mathematics and ethnomathematics.To find the volume and surface area of Damaru, first let us derive formula for calculating surface area and volume of frustum of cone.

For Surface Area of Frustum



In Similar Triangles ABE and

$$\frac{R}{r}\frac{R}{r} = \frac{L}{l_1}\frac{L}{l_1}$$

Or, $\frac{R}{r} = \frac{l_1+l}{l_1}\frac{R}{r} = \frac{l_1+l}{l_1}$

Or, $Rl_1\, Rl_1 = rl_1\, rl_1 + rl\, rl$

Or, $Rl_1\, Rl_1 - rl_1\, rl_1 = rl\, rl$

∴ ∴, $l_1\, l_1 = \frac{lr}{R-r}\frac{lr}{R-r}$……………….(i)

Slant height of cone L = $l + l_1\; l + l_1$

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Or, L = $l + \frac{lr}{R-r}$ $l + \frac{lr}{R-r}$ $\therefore L = \frac{l(R-r+lr)}{R-r}$ $\therefore L = \frac{l(R-r+lr)}{R-r}$ …………(ii)

Now,

CSA of Frustum = $\pi RL - \pi r l_1$ $\pi RL - \pi r l_1$ ………………(iii)

Using (i), (ii) and (iii) we get

Curve Surface Area (CSA) of Frustum = $\frac{\pi l\{R^2 - r(r+R-l)\}}{R-r}$ $\frac{\pi l\{R^2 - r(r+R-l)\}}{R-r}$

For the Volume of Frustum

In Similar Triangles ABE and ACD , $\frac{R}{r} = \frac{H}{h_1}$ $\frac{R}{r} = \frac{H}{h_1}$

Or, H= $\frac{R}{r} \times h_1$ $\frac{R}{r} \times h_1$ …………………(a)

In right angled triangle ABE

h_1 h_1 = $\sqrt{(l_1^2 - r^2)}$ $\sqrt{(l_1^2 - r^2)}$

By using (i) we get

$\therefore\ h_1$ $\therefore\ h_1$ = $\sqrt{\left\{\left(\frac{lr}{R-r}\right)^2 - r^2\right\}}$ $\sqrt{\left\{\left(\frac{lr}{R-r}\right)^2 - r^2\right\}}$ …………………………..(b)

Using (a) and (b) we get

H = $\sqrt{\left\{\left(\frac{lr}{R-r}\right)^2 - r^2\right\}}$ $\sqrt{\left\{\left(\frac{lr}{R-r}\right)^2 - r^2\right\}}$ $\times \frac{R}{r}$ $\times \frac{R}{r}$ ……..………………..(c)

The volume of frustum = $\frac{1}{3}\pi R^2 H - \frac{1}{3}\pi r^2 h_1$ $\frac{1}{3}\pi R^2 H - \frac{1}{3}\pi r^2 h_1$

Using (b) and (c)

The volume of frustum = $\frac{1}{3}\pi$ $\frac{1}{3}\pi$ $\sqrt{\left\{\left(\frac{l}{R-r}\right)^2 - 1\right\}}$ $\sqrt{\left\{\left(\frac{l}{R-r}\right)^2 - 1\right\}}$ $\times (R^3 - r^3)$ $\times (R^3 - r^3)$

Total Surface Area of Damaru = 2

$\times\, C.S.A\ of\ frustum + C.S.A.of\ cylinder + 2 \times area\ of\ circle$

$\times\, C.S.A\ of\ frustum + C.S.A.of\ cylinder + 2 \times area\ of\ circle$

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$= 2 \times \times \frac{\pi l\{R^2 - r(r+R-l)\}}{R-r} \frac{\pi l\{R^2 - r(r+R-l)\}}{R-r} + 2\pi rh\, \pi rh +$

$2\pi r^2\, 2\pi r^2$

∴ ∴ Total Surface Area of Damaru = $2\pi\left[\frac{l\{R^2 - r(r+R-l)\}}{R-r} + rh + r^2\right]$

$2\pi\left[\frac{l\{R^2 - r(r+R-l)\}}{R-r} + rh + r^2\right]$

Volume of Damaru = 2 $\times\, volume\ of\ frustum\ + Volume\ of\ cylinder$

$\times\, volume\ of\ frustum\ + Volume\ of\ cylinder$

$= 2 \times \frac{1}{3}\pi \times \frac{1}{3}\pi \sqrt{\left\{\left(\frac{l}{R-r}\right)^2 - 1\right\}} \sqrt{\left\{\left(\frac{l}{R-r}\right)^2 - 1\right\}} \times (R^3 - r^3)$

$\times (R^3 - r^3) + \pi r^2 h\, \pi r^2 h$

∴ ∴ Volume of Damaru = $2 \times \frac{1}{3}\pi \times \frac{1}{3}\pi \sqrt{\left\{\left(\frac{l}{R-r}\right)^2 - 1\right\}} \sqrt{\left\{\left(\frac{l}{R-r}\right)^2 - 1\right\}} \times (R^3 - r^3)$

$\times (R^3 - r^3) + \pi r^2 h\, \pi r^2 h$

Where, h= height of cylindrical part of Damaru

l = slant height of frustum part of Damaru

R, r = radii of circular section of Damaru

From an outsider's perspective, the Damaru can be mathematically modeled using geometric principles embedded in its construction. The instrument's structure, consisting of two truncated cones joined by a central cylinder, can be described using specific mathematical formulas for surface area and volume. The total surface area of the Damaru is derived by combining the curved surface areas of the frustums, the cylindrical section, and the circular ends. The volume, on the other hand, is calculated by summing the volumes of the two frustums and the cylinder. By applying these mathematical principles, one can see the precision and sophistication inherent in the Damaru's design. For instance, with given values for the radii *R* and *r*, slant height *l*, and the height *h* of the cylindrical section, one can accurately compute the Damaru's volume and surface area. This mathematical modelling not

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only mirrors the artisans' implicit knowledge but also serves as a powerful example of how traditional craftsmanship can embody complex mathematical concepts, providing rich learning opportunities in a classroom setting.

The construction of the *Damaru* by the Chundara people illustrates a profound and nuanced integration of mathematical knowledge and practices, deeply embedded in their cultural and craft traditions. The process, while rooted in practical necessity, demonstrates an implicit mastery of various mathematical concepts that are often unrecognized outside their cultural context. Chundara's approach to building the *Damaru* involves a sophisticated understanding of measurement, proportions, and geometry. They use traditional units like handspans and digits (amal) to gauge the size and shape of the timber, reflecting a deep-seated comprehension of proportionality and scaling. These non-standard measurements are not arbitrary but are carefully calibrated to ensure that the final product meets precise functional requirements. Their ability to select and shape timber based on these measurements reveals an intuitive grasp of geometric principles such as volume and surface area, which are crucial for achieving the desired resonance and structural integrity of the drum.

In shaping the *Damaru*, the Chundara exhibit an implicit understanding of conical and cylindrical geometry. The drum’s double-conical form, with its symmetrical and uniform hollow, showcases their skill in applying geometric transformations to create a functional and aesthetically pleasing instrument. This process involves not only the crafting of the outer shape but also the internal dimensions, ensuring that the hollow section is smooth and consistent to produce the correct sound. The precise carving and shaping techniques highlight their familiarity with concepts of symmetry and uniformity, essential for achieving the drum's acoustic properties. The Chundara’s practices reflect a rich repository of ethnomathematical and ethnoscientific knowledge. Their craftsmanship is a blend of practical experience and cultural tradition, passed down through generations.

This knowledge encompasses not only the techniques of shaping and carving but also a deep understanding of material properties. The selection of timber based on its resonance and suitability for crafting the *Damaru* demonstrates an application of acoustics and material science within their cultural framework. Moreover, the collaborative nature of the *Damaru* construction process illustrates a community-based approach to learning and knowledge

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transmission. The skills and techniques involved are shared and refined within the community, emphasizing the role of collective experience and oral tradition in preserving and evolving their mathematical and scientific knowledge. This community-based learning reflects an ethnoscientific approach where practical knowledge is continually adapted and improved through hands-on experience and shared expertise.

Recognizing and integrating these implicit mathematical practices into modern educational contexts can enrich our understanding of mathematics as a culturally embedded discipline. Chundara's methods provide valuable insights into how traditional knowledge systems can inform and complement formal mathematical education. By acknowledging and valuing such ethnomathematical knowledge, educators can bridge the gap between indigenous practices and academic mathematics, offering students a more inclusive and diverse perspective on mathematical concepts. In essence, the construction of the *Damaru* by the Chundara people exemplifies how traditional practices encapsulate complex mathematical and scientific ideas. Their ethnomathematical knowledge and thinking, while implicit, reveal a deep connection between cultural practices and mathematical understanding, showcasing the richness of their ethnoknowledge and ethnoscience.

Concluding Remarks

The construction of the Damaru by the Chundara people is a testament to the intricate and sophisticated mathematical knowledge embedded within their cultural practices. This process, which might seem purely artisanal or traditional at first glance, reveals a rich tapestry of implicit geometrical concepts and measurements. By using non-standard units such as handspans and digits (amal), the Chundaras effectively gauge and shape the timber, demonstrating an intuitive grasp of proportionality, symmetry, and volume. These traditional methods, passed down through generations, embody a form of ethnomathematics that seamlessly integrates cultural heritage with practical craftsmanship.

The mathematical principles inherent in the Damaru's construction – such as the understanding of cylindrical and conical geometry, the importance of uniformity and balance, and the relationship between material properties and acoustics – highlight the Chundara people’s expertise in both ethnoknowledge and ethnoscience. Their ability to produce a harmonious and functional musical instrument through these practices underscores the sophistication of their traditional knowledge systems. This integration of cultural practices with mathematical thinking not only enriches our understanding of mathematics but also challenges the conventional boundaries of mathematical education.

Recognizing the mathematical practices of the Chundara and similar cultural groups offers valuable insights into the diverse ways in which mathematical knowledge can be conceptualized and applied. By integrating these ethnomathematical concepts into modern educational frameworks, educators can provide a more inclusive and contextually relevant learning experience. This approach not only honors the cultural heritage of indigenous and traditional communities but also enriches the mathematical understanding of students by connecting abstract concepts with tangible, real-world applications. The ethnomathematical practices of the Chundara people, as exemplified in the construction of the Damaru, highlight the profound connection between cultural traditions and mathematical knowledge.

These practices reveal a deep, albeit implicit, understanding of geometry, measurement, and material science, demonstrating the richness of their ethnoknowledge and ethnoscience. By acknowledging and incorporating these traditional mathematical practices into contemporary education, we can bridge cultural gaps and foster a more holistic understanding of mathematics as a discipline deeply rooted in human culture and experience. In conclusion, my study gave me the insight that they used lots of mathematical concepts and knowledges in their work. The mathematical concepts and knowledge such as length, breadth, height, area, volume, circle, diameter, perimeter, parallel planes, cylinder, cone, rectangular parallelepiped, rotation, axis of transformation, hyperboloid, hemisphere and more are found in their daily activities and practices. It was also found that they had a high degree of estimation in selecting timber to make particular shape and size for required volume of the wooden stuffs. All these knowledges is derived from their ancestors outside schooling but it is factual, scientific and a strong base for their work and living.

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